COMMON PRE-BOARD EXAMINATION 2022-23

Subject: MATHEMATICS (041)

Date: Duration: 3 Hours

TOTAL MARKS: 80

General Instructions:

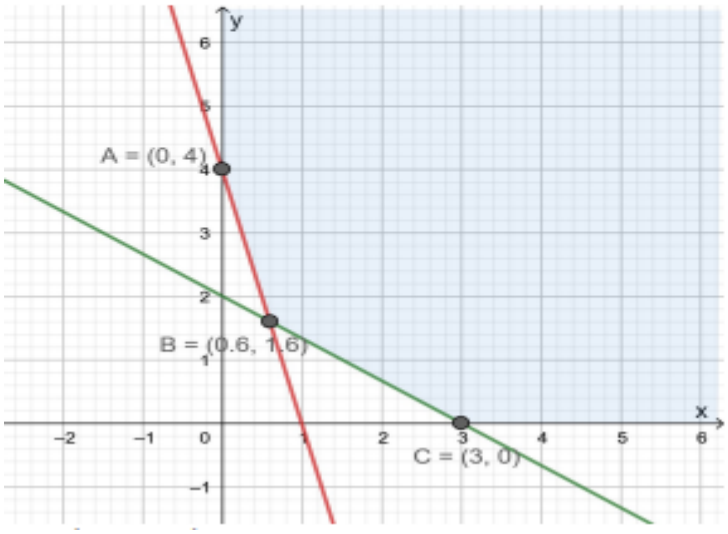
1. This question paper contains **five sections A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **2 Assertion- Reason** based questions of 1 mark each.
3. **Section B** has **5 Very Short Answer(VSA)** type questions of 2 marks each.
4. **Section C** has **6 Short Answer (SA)** type questions of 3 marks each.
5. **Section D** has **4 Long Answer (LA)** type questions of 5 marks each.
6. **Section E** has **3 source based/ case based/ passage based** integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions- Each question carries 1 mark)

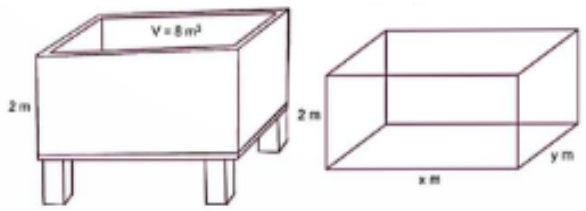
Q1	A is a square matrix such that $A^2 = I$, where I is the identity matrix, then find the value of $(A-I)^3 + (A+I)^3 - 7A$ a) I b) –I c) A d) –A	1
Q2	A is a 3x3 invertible matrix, then what will be value of k if $det(A^{-1}) = (detA)^k$ a) 1 b) -1 c) 2 d)-2	1
Q3	If $\vec{a}$ is a unit vector such that $(2\vec{x} - 3\vec{a}).(2\vec{x} + 3\vec{a}) = 91$, then find $\lvert\vec{x}\rvert$ a) 4 b) 5 c) 6 d) 10	1
Q4	For what value of k, the following function is continuous at x = 0? $f(x) = \begin{cases} \frac{1-cos4x}{8x^2} & if\ x \neq 0 \\ k & if\ x=0 \end{cases}$ a) 1 b) -1 c) ½ d) – ½	1

Q5	Evaluate $\int_0^{\pi/2} \frac{sinx-cosx}{1+sinxcosx}\, dx$ a) π b)$\pi/2$ c) 1 d) 0	1
Q6	Write the integrating factor of the differential equation $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$ a) $\tan^{-1} x$ b) e^x c)$e^{\tan^{-1} x}$ d) $\frac{1}{1+x^2}$	1
Q7	The solution set of the inequality 3x + 4y < 4 is a) an open half plane not containing origin b) an open half plane containing origin c) the whole XY [plane not containing the line 3x+4y = 4 d) the whole XY plane containing the origin	1
Q8	Find λ when the projection of $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}\ on\ \vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\ is\ 4$ units a) -5 b) -4 c) 4 d) 5	1
Q9	If $\int \left(\frac{x-1}{x^2}\right) e^x = f(x)e^x + \ C, then\ write\ the\ value\ of\ f(x)$ a) x b) 1/x c) x^2 d) x-1	1
Q 10	If matrix A = $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ $and\ A^2 = p\ A$, write the value of p. a) 2 b) -2 c) 1 d) 4	1
Q11	Let A and B be two events. If P(A) = 0.2, P(B) = 0.4 and P(AUB) = 0.5, then find P(A/B) a) 0 b) ½ c) ¼ d) 1	1
Q12	If A is a square matrix of order 3 such that $\|adjA\| = 81,$ then find $\|A\|$. a) ∓ 81 b) ∓ 9 c) ∓ 27 d) ∓ 3	1
Q13	Evaluate $\begin{vmatrix} cos15 & sin15 \\ sin75 & cos75 \end{vmatrix}$ a) 0 b) 1 c) -1 d) ½	1
Q14	Write the sum of order and degree of the differential equation $\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\}$ = 0 a) 3 b) 4 c) 5 d) 2	1

Q15	. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at  (a)(0.6, 1.6) *only* (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only (d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)	1
Q16	What is $\frac{dy}{dx}$ at x = 2 if x – y = k. a) 0 b) 1 c) -1 d) 2	1
Q17	If $\|\vec{a}\| = 4$, $\|\vec{b}\| = 3$ *and* $\vec{a}.\vec{b} = 6\sqrt{3}$, then value of $\|\vec{a} \times \vec{b}\|$. a) 4 b) 3 c) 6 d)12	1
Q18	If a line makes angles α, β *and* γ with the coordinate axes, write the value of $sin^2\alpha + sin^2\beta + sin^2\gamma$ a) 1 b)2 c) 3 d)0	1
	In Q19 and Q20, a statement of Assertion, A is followed by a statement of Reason, R is given. Mark the correct choice.	
Q19	**Assertion (A)** : $\sin^{-1}\left(sin\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$ **Reason (R)** : $\sin^{-1}(sinx) = x\ if\ x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ a) Both A and R are true and R is the correct explanation of A b) Both A and R are true but R is not the correct explanation of A c) A is true but R is false d) A is false but R is true	1

Q20	**Assertion (A)** : The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-3}$ is 90°. **Reason (R):** Skew lines are lines in different planes which are parallel and intersecting. a) Both A and R are true and R is the correct explanation of A b) Both A and R are true but R is not the correct explanation of A c) A is true but R is false d) A is false but R is true	1
	SECTION B (2 Marks each)	
Q21	Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$ **OR** Let f: N → N be defined by f(x) = $f(x) = \begin{cases} \frac{n+1}{2}, & if\ n\ is\ odd \\ \frac{n}{2}, & if\ n\ is\ even \end{cases}$. For all n ∈ N, state whether f is bijective.	2
Q22	A particle moves along the curve $6y = x^3 + 2$. Find the x – coordinate of the points on the curve at which the y – coordinate is changing 8 times as fast as x – coordinate.	2
Q23	If $\vec{a}\ and\ \vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ if $\vec{a} - \sqrt{2}\vec{b}$ be a unit vector. **OR** If the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{2z-6}{4}$ and $\frac{1-x}{-3\lambda} = \frac{y-1}{1} = \frac{10z-12}{-10}$ are perpendicular to each other, find the value of λ	2
Q24	If $y = logtan\left(\frac{\pi}{4} + \frac{x}{2}\right), then\ show\ that\ \frac{dy}{dx} = secx.$	2
Q25	Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$, and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}\ and\ \vec{p}.\vec{c} = 18.$	2
	SECTION C (3 Marks each)	
Q26	Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)}dx$	3
Q27	A problem in Mathematics is given to three students whose chances of solving it are ½, 1/3 and ¼. If all of them try to solve the problem, what is the probability that (i) problem is solved (ii) exactly one of them will solve.	3

	OR There are 4 cards numbered 1 to 4, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards . Find the mean value of X.	
Q28	Evaluate $\int_{-1}^{2}\|x^3 - x\|dx$ **OR** Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{cotx}}$	3
Q29	Solve the differential equation $x\frac{dy}{dx}sin\left(\frac{y}{x}\right) + x - ysin\left(\frac{y}{x}\right) = 0$ **OR** Solve the differential equation $(1 + x^2)\frac{dy}{dx}$+ y = $\tan^{-1} x$	3
Q30	Solve the following Linear Programming Problem graphically: Maximize Z = 400x + 300y subject to the constraints x + y ≤ 200 , x ≤ 40 , x ≥ 20 and y ≥ 0.	3
Q31	Evaluate $\int\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$	3
	SECTION D (5 Marks each)	
Q32	Make a rough sketch of the region $\{(x, y): 0 \le y \le x^2, 0 \le y \le x, 0 \le x \le 2\}$ and find the area of the region using integration.	5
Q33	Let R be the relation defined on the set of natural numbers N as R = $\{(x, y): x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify R is reflexive, symmetric or transitive. **OR** The relation R in the set N x N is defined as follows: For (a, b) and (c, d) ∈ N x N, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation on N x N	5
Q34	Find the shortest distance between the lines $\vec{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \alpha(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ And $\vec{r} = -\hat{\imath} - \hat{\jmath} - \hat{k} + \beta(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$. **OR**	5

	Find the vector and Cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ *and* $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.	
Q35	Solve the following system of linear equations using matrix method: x – y +2z = 7, 3x + 4y – 5z = -5, 2x – y + 3 z = 12	5
	Q36, Q37 and Q38 are Case Based Questions	
Q36	. **Case-Study 1**: Read the following passage and answer the questions given below.  The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6, 0 \le x \le 12$, m being a constant, where f(x) is the temperature in °F at x days. (i) Is the function differentiable in the interval (0, 12)? Justify your answer. 1 (ii) If 6 is the critical point of the function, then find the value of the constant m. 1 (iii) Find the intervals in which the function is strictly increasing/strictly decreasing. 2 OR (iii) Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function. 2	
Q37	**Case Study – 2**  Read the text carefully and answer the questions: On the request of villagers , a construction agency designs a tank with the help of an architect. The tank consists of a rectangular base with rectangular sides open at the top so that its depth is 2m	

	and volume is 8 m^3 as shown above. The construction of the tank costs Rs 70 per square m for the base and Rs 45 per square m for sides. (i)Express making cost C in terms of length of rectangle base. (ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables. (iii) Find the value of x so that the cost of construction is minimum.	 1 1 2
Q38	**Case Study -3**  Inan office three employees, Govind, Priyanka and Tasneem , process incoming copies of a certain forms.Govind processes 50% of the forms, Priyanka processes 20% and Tasneem the remaining 30 % of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tasneem has an error rate of 0.03. (i)The Manager of the company wants to do the quality check, During the inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind. (ii) Find the probability that Priyanka processed the form and committed an error.	 2 2